# Mathematical model of temperature field and mushy zone position of continuous ingot

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## Summary

The mathematical model of convective-conductive heat transfer in a continuous billet was developed to simulate the temperature distribution and the boundary position of phase changes in the ingot during the casting process. The model takes into account the temperature dependence of metal's thermal properties and the heat transfer with moving metal. The heat transfer in an ingot and the heat exchange inside the walls of the mold are described by nonlinear unsteady partial differential equations. The boundary conditions assigned for the part of the ingot inside the mold correspond to the nature of heat transfer under the slag during casting. Also they reflect the fact that a gap filled partially with encrusted slag and partially with gas located between the ingot surface and the wall of the mold. The unknown boundary between liquid and solid phases is considered as a mushy zone given by the condition of temperature equality and the Stefan condition. A finite-difference method was used to numerical solve the problem. The analysis of the qualitative behaviour of the mushy zone was held. The results of calculations of the influence of variations of the casting speed, secondary cooling water rate and thermal parameters on the depth and shape of the mushy zone are received. These results can be used in the future to assess the adequacy of the mathematical model of temperature field of continuous casting ingot and to develop an automatic control system of casting machine.

Key Words: Continuous casting, mathematical modelling, temperature field, phase-change boundary, mushy zone, metallurgical length, liquid pool, shell dynamics

## Introduction

It is well known that the properties of continuous casting (CC) production are highly dependent on temperature dynamics during the casting process. The control of the CC process based on strand temperature distribution is a complex problem and cannot be achieved without proper knowledge of heat and mass transfer and solidification dynamics. Conducting of industrial trials is too expensive. Development of computer technology has made computer experiments the main tool of metallurgical processes investigation.

Computer simulation based on various mathematical models. For designing continuous casting machines (CCM), building automatic control systems (ACS), and developing new casting technologies — and in turn for developing a mathematical model to adequately achieve these three goals, it is necessary to study how one of the casting parameters affects others. So we have to choose or to develop a model which takes into account certain conditions.

For instance, if we want to increase the productivity of CCM, we need to increase the casting speed. This raises the problem of determining of the metallurgical length. In other words, we need to know at what distance from meniscus crystallization is fully completed.

In this paper the mathematical model of temperature field and mushy zone position of continuous ingot is presented. For its numerical solution the finite-differences method is used. The results of calculations of the influence of variations of the casting speed, secondary cooling water rate and thermal parameters on the depth and shape of the mushy zone are received and also presented here.

## State of the Art

One of the simplest methods of the phase-change position determination is the engineering method of the square root [1], which in some cases gives a sufficiently close to the reality of the data. Nevertheless, it cannot be always considered as enough reliable. So the next step in improvement of mathematical model of continuous ingot crystallization is the consideration of Stefan condition. In [2], the classical and the generalized formulation of Stefan problem are given, as well as the basic mathematical results on the existence and uniqueness of analytical solutions are presented. The numerical solution of this problem was a purpose of the work a lot of researchers. Many authors use the so-called method of spreading [3], which consists in the introduction of the Dirac delta function with the aim of defining the heat transfer inside a continuous medium.

Then the Dirac delta function is replaced of a simple approximation. At the same time the thermal physical characteristics also are replaced of some effective values.

But recent researches showed that in reality there is not so called jump of the thermal physical parameters at the phase-change boundary.

Actually to deal with the high variability of the described problem the heat and mass phenomenon must also be taken into account, and therefore the employed numerical model has to cover all the phase and structural changes [4]. A temperature field of steel slab in continuous casting process must be described as a heat transfer problem involving solidification. The phase trans-formation phenomenon is dominant in such technological process.

In [5] the 2-D mathematical model examines the two-dimensional temperature field and the phase boundary in a longitudinal section of a wide slab. In [6] the similar 3-D mathematical model is proposed. Both of the models account for the complex geometry of the secondary cooling zone (SCZ), the location of the nozzles in the SCZ, the dependence of heat transfer on water discharge in the nozzles, and the dependence of the thermophysical parameters on the temperature of the metal. But both of them don't consider the mushy zone between solid and liquid phases.

In [7] heat transfer in an ingot in pulling it through the crystallizer in the process of continuous casting of steel is modeled by two different methods: in the classical formulation of the Stefan problem and on condition that a two-phase buffer zone exists between the liquid core of the ingot and its solid shell. A comparison of the obtained results is made using a standard slab crystallizer as an example.

In this paper another approach to simulate the liquidus and solidus behavior is considered.

## The Mathematical Model

Mathematical model of convective-conductive heat transfer in a continuous billet based on the mathematical model proposed in [6]. It accounts for the dependence of the thermophysical characteristics of metal on temperature. That's why convective-conductive heat transfer in an ingot and heat exchange inside the walls of the mold are described by nonlinear unsteady partial differential equations. The boundary conditions assigned for the part of the ingot inside the mold correspond to the nature of heat transfer under the slag during casting. Also they reflect the fact that a gap filled partly with encrusted slag and partly with gas is located between the ingot surface and the wall of the mold. The coordinate system is tied to the continuous caster (Fig. 1) and has its origin at the level of the meniscus. The equation of convective-conductive heat transfer inside the ingot is

$$\frac{\partial T}{\partial \tau}+v(\tau)\frac{\partial T}{\partial z}=$$
$$=\frac{1}{c(T)\rho(T)}\left\{\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(\lambda(T)\frac{\partial T}{\partial z}\right)\right\},$$

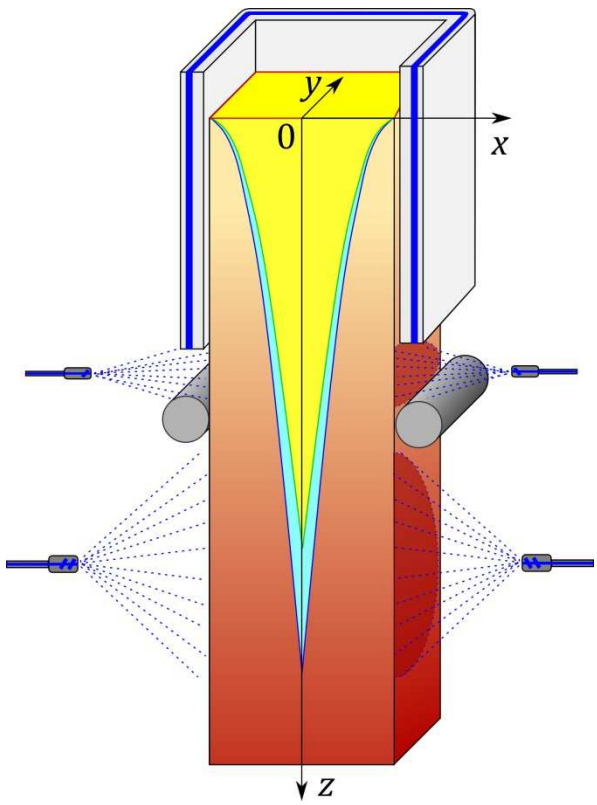


*Figure 1: The coordinate system in the area modeled*

where $\tau$ is time; $T = T(\tau; x; y; z)$ is temperature; $v(\tau)$ is the speed of the ingot; $c(T)$ is heat capacity; $\rho(T)$ is density; and $\alpha(T)$ is the thermal conductivity of the metal being cast.

Unknown boundary position between liquid phase and mushy zone $x = l(z)$ is determined from the temperature-equality condition and the Stefan condition

$$T(x,z)\big|_{l+}=T(x,z)\big|_{l-}=T_l\ ,$$

$$\lambda(T)\frac{\partial T}{\partial \bar{n}}\bigg|_{l_-}-\lambda(T)\frac{\partial T}{\partial \bar{n}}\bigg|_{l_+}=\mu_l\rho_l\left(v(\tau)\frac{\partial l}{\partial z}+\frac{\partial l}{\partial \tau}\right).$$

where $n$ is a normal to the phase boundary; $\mu_l$ is the latent heat of partially crystallization; $T_l$ is the liquidus temperature; $\rho_l=\rho(T_l)$; and $l$ is the liquid phase – mushy zone boundary.

And boundary position between mushy zone and solid phase $x = s(z)$ is determined from the analogous conditions

$$T(x,z)\big|_{s+}=T(x,z)\big|_{s-}=T_s\ ,$$

$$\lambda(T)\frac{\partial T}{\partial \bar{n}}\bigg|_{s_-}-\lambda(T)\frac{\partial T}{\partial \bar{n}}\bigg|_{s_+}=(\mu-\mu_l)\rho_s\left(v(\tau)\frac{\partial s}{\partial z}+\frac{\partial s}{\partial \tau}\right),$$

where $\mu$ is the latent heat of crystallization; $T_s$ is the solidus temperature; $\rho_s = \rho(T_s)$ ; and $s$ is the mushy zone – solid phase boundary.
The boundary conditions in the SCZ account for the complex heat transfer mechanism resulting from convection and radiation:

$$-\lambda(T)\frac{\partial T}{\partial x}\bigg|_{x=d} =$$
$$= \alpha(G_m(\tau), z)(T_{Am} - T|_{x=d}) + C_m(T_{Am}^4 - (T|_{x=d})^4),$$

$$-\lambda(T)\frac{\partial T}{\partial y}\bigg|_{y=d} =$$
$$= \alpha(G_m(\tau), z)(T_{Am} - T|_{y=d}) + C_m(T_{Am}^4 - (T|_{y=d})^4),$$

where $G_m(\tau)$, $C_m$, *and* $T_{Am}$ are respectively the water discharge, the corrected value of the radiation coefficient on the surface of the ingot, and the ambient temperature in the *m*-th section of the SCZ; *l* is half the thickness of the ingot: *x*= *d* is a point on the surface of the ingot; and $\alpha(G_m(\tau), z)$ is the heat-transfer coefficient on the surface of the ingot. We assigned initial conditions for the temperature field and the position of the phase boundary. The nonlinear boundary-value problem is solved by the finite-differences method. Unknown boundary position was calculated by the method described in [8].
The proposed mathematical model has been adapted to the conditions which actually exist during the casting process on the machine No2 of Yenakiieve Iron and Steel Works Metinvest Holding (EMZ). Algorithms were described in [9] for adjusting the parameters of the model in accordance with incoming real-time information obtained on ingot surface temperature from measuring instruments.
The model makes it possible to observe the dynamics of the ingot solidification process while varying the input parameters. The output information of the model consists of the temperature of the metal and the position of the mushy zone boundaries in a longitudinal section of the slab at any chosen moment of time. Also, this information can be used to calculate temperature gradients, heat removal, and the average temperature of the ingot surface at the centre of its broad face in different cross sections.
The basic regime is: 0.2 m thickness of the slab, an ingot withdrawal speed of 3.5 m/min. Thermal physical parameters for steel the chemical composition is presented in Table 1, the temperature of the incoming melt to 1550° C. Mold and secondary cooling modes correspond to the technological instructions. The results of phase-change position calculation are presented on Fig.2.

*Table 1. Steel the chemical composition*

| | |
|---|---|
| C | 0,36…0,44 |
| Si | 0,17…0,37 |
| Mn | 0,5…0,8 |
| Ni | < 0,3 |
| S | < 0,035 |
| P | < 0,035 |
| Cr | 0,8…1,1 |
| Cu | < 0,3 |

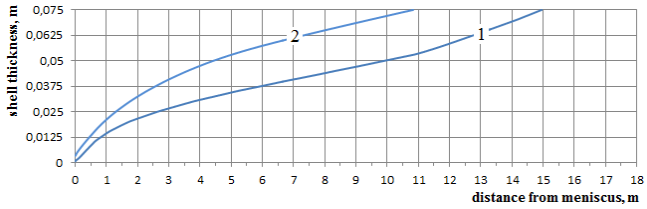


*Figure 2: The solidus (1) and liquidus (2) at the casting speed 3.5m/min*

## Mushy zone depending on casting speed

During casting process it is recommended to maintain an ingot withdrawal speed at a certain technologically specified level. However, for technical reasons, the speed can be varied. For instance it is needed to reduce the normative speed for a short time for the necessary technological operations (replacing the submerged entry nozzle, etc.). When casting speed changes the depth of the liquid pool changes too. Nowadays researchers made attempts to develop algorithms for control the depth of the liquid phase by adjusting the water flow rates in the secondary cooling zone [4]. To solve this problem it is need to know how much the casting speed effects on the form of solidus phase-change boundary, and how much the secondary cooling modes effect on it.
Fig. 3 is a graph showing the position of the mushy zone at different speeds of the ingot withdrawal. The greatest response observed in the point of final solidification, i.e. depth of the liquid phase depends significantly on the ingot withdrawal speed.

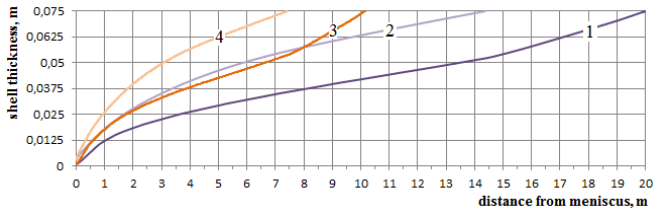


*Figure 3: The solidus (1) and liquidus (2) at the casting speed 4.5m/min and the solidus (3) and liquidus (4) at the casting speed 2.5m/min (at the same cooling regime)*

Since for the productivity increasing the information about solidus position is more important, we will pay

our attention on the phase-change boundary between mushy zone and solid metal below.
Fig.4. show us how much the casting speed effects on the form of solidus phase-change boundary at the same secondary cooling regime.

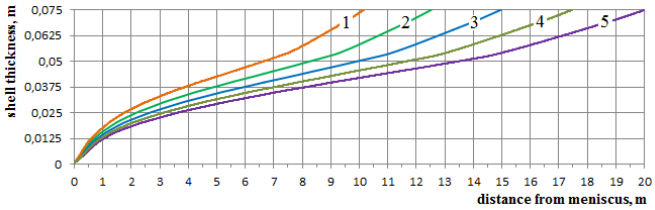


*Figure 4: The solidus for 2.5m/min (1), 3.0m/min (2), 3.5m/min (3), 4.0m/min (4), and 4.5m/min (5) at the same secondary cooling regime.*

## The liquidus position depending on secondary cooling mode

Control parameters in the secondary cooling system are the discharges of cooling water in the various sections. One of the major challenges is to control the depth of the mushy zone position at a casting speed changes. Potential possibilities of such control are presented at the Fig.5. We can see that the possibility of the metallurgical length control by the secondary cooling under the changes in casting speed is rather limited. However, it should be noted that at more high casting speeds the final crystallization boundary form and depth adjustment available in a wider range.

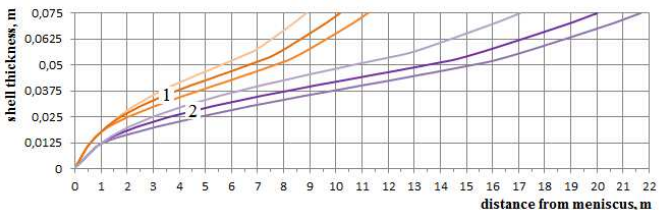


*Figure 5: The solidus at the secondary regime variations from min to max for casting speed 2.5m/min (1), and 4.5m/min (2)*

## The effects of thermal characteristics on simulation results

To mathematical model could be used in ACS CCM it is needed to guarantee a certain precision of the real process displaying. As already mentioned above, it is the tasks of identification systems theory. Since most uncertain may have thermal characteristics, it is necessary to establish the degree of influence of the errors of their determination of the simulation results.
Experiments show the deviation liquid pool form when an error in the determination of the thermal conductivity of the cast metal component of approximately ± 10%. The depth of the liquid phase thus also varies in the range of about ± 9%.
Analysing the nature of entering into the heat equation other thermophysical properties (density and specific heat), we can conclude that the error in their definition will affect the calculations in a similar manner.
Studies of the effect of temperature changes in coming melt were also carried out. Calculations showed that the temperature deviation metal poured into the mold, within ±10°C is negligible effect on the depth and shape of the liquid phase.

## Conclusion

The mathematical model of temperature field of continuous ingot is developed. The numerical experiments with different casting parameters were held. The dynamics of mushy zone under various control actions and disturbances is studied. The results of calculations of the influence of variations of the casting speed, secondary cooling water discharge and thermal parameters on the depth and shape of the liquid pool are presented. The simulation results show the limit of the possibility of controlling the liquid phase depth at a casting speed changes by secondary cooling.

## Abbreviations

| | |
|---|---|
| ACS | |
| CCM | Continuous casting machine |
| SCZ | Secondary cooling zone |

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